THE COLLEGES OF OXFORD UNIVERSITY

PHYSICS

Wednesday 5 November 2008

Time allowed: 2 hours

For candidates applying for Physics, and Physics and Philosophy

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There are two parts (A and B) to this test, carrying equal weight.

Attempt as many questions as you can from each part. Write on white A4 lined paper. At the end of the test put your answer sheets in order and collate with a treasury tag. Attach a cover sheet to your answers. **DO NOT** write your name anywhere except on the cover sheet.

Marks for each question are indicated in the right hand margin. There are a total of 100 marks available and total marks for each section are indicated at the start of a section. You are advised to divide your time according to the marks available, and to spend equal effort on parts A and B.

No calculators, tables or formula sheets may be used.

Answers in Part A should be given exactly unless indicated otherwise. Numeric answers in Part B should be calculated to 2 significant figures.

Use $g = 10\,\mathrm{m\,s^{-2}}$.

Do NOT turn over until told that you may do so.

Part A: Mathematics for Physics [50 Marks]

Answers in Part A should be given exactly unless indicated otherwise.

1. Evaluate the sum of integers $1 + 2 + 3 + \cdots + 99 + 100$. [3]

2. Evaluate $(0.25)^{-1/2}$ and $(0.09)^{3/2}$ [4]

3. The first three terms of the series expansion of $(1 + x)^m$ are:

$$1 + mx + \frac{m(m-1)x^2}{2}.$$

Find the first three terms in the series expansion of $(1+x)^{m+1}(1-2x)^m$. [5]

4. Find the set of values of x for which

$$\frac{x^2 + 2}{1 - x^2} < 3.$$ [3]

5. Given that $x = \log_9 2$, find, in terms of x, (i) $\log_2 9$; (ii) $\log_8 3$. [4]

6. Find the two values of x for which $1, x^2, x$ are successive terms of an arithmetic progression. [3]

7. Determine the value of a such that the curve $y = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots$ and the line $y = ax$ have the same gradient at $x = 0$. What value will a have if instead they have the same gradient at $x = \frac{1}{4}$?. [4]

8. The points (5,2) and $(-3, 8)$ are at opposite ends of the diameter of a circle. Determine the equation of the circle. [4]

9. A die is biased so that the numbers 5 and 6 are obtained three times as often as 2, 3 and 4, and the number 1 is never obtained. Calculate the probability that (i) a two is thrown; (ii) two consecutive throws give a total ≥ 10. [5]

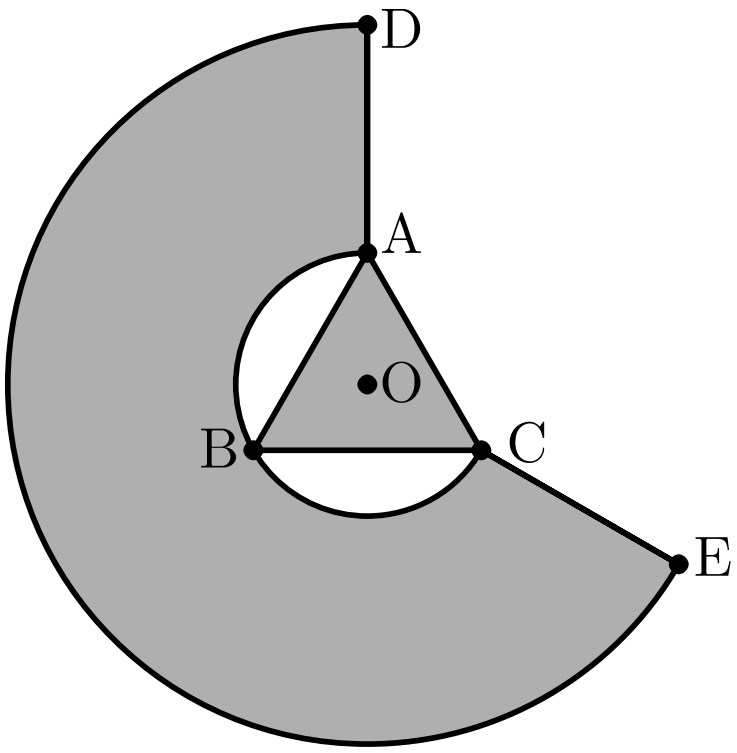
10. A cube has side a. Find the length of its body diagonal. [3]

11. Evaluate

    (i) $\int_{-1}^{1} x + x^3 + x^5 + x^7 \, \mathrm{d}x$

    (ii) $\int_{0}^{1} \frac{x^9 + x^{99}}{11} \, \mathrm{d}x$. [4]

12. In the figure below, the shaded area ADEC is defined by concentric circles which share a common centre O with the shaded triangle ABC. The straight lines AD and CE, if extended, pass through O. The lengths AD=AB=BC=CA=CE. Find the ratio of the two shaded areas ADEC and ABC. [8]



Part B: Physics [50 Marks]

Numeric answers in Part B should be calculated to 2 significant figures. Use $\mathbf{g = 10\,m\,s^{-2}}$.

Multiple choice (10 marks)

13. A symmetric seesaw is 3 m long from end to end. If a boy of mass 20 kg sits on one end, how far away from him should a girl of mass 30 kg sit to balance the seesaw?

A 0.5 m **B** 1.0 m
C 2.0 m **D** 2.5 m [1]

14. When nuclear fission occurs in a commercial nuclear reactor the mass of the products compared with the mass of the reactants is

A increased
B decreased
C stays the same
D it depends on the reaction [1]

15. The visible universe contains about 400 billion galaxies (where 1 billion equals 10^9). Our galaxy contains about 250 billion stars. The mass of our sun is about 2×10^{30} kg. NASA estimates that dark matter out-masses stars by about 20:1. Use this data to estimate the total mass of the visible universe.

A 4.2×10^{36} kg **B** 9.5×10^{51} kg
C 2.0×10^{53} kg **D** 4.2×10^{54} kg [1]

16. A solar eclipse can only occur when the moon's phase is

A new moon **B** full moon
C waning **D** waxing [1]

17. When an ideal gas is heated in a container of fixed volume then

A the pressure and density both rise
B the pressure rises and the density falls
C the pressure rises and the density stays the same
D the pressure stays the same and the density falls [1]

18. A physics lecture theatre is situated 3 m east and 4 m above reception. Calculate the minimum energy a 60 kg receptionist would have to expend to reach the lecture theatre.

A 1800 J **B** 2400 J
C 3000 J **D** 4200 J [1]

19. A 3.6 V mobile phone battery can produce 0.7 A of current for 1 hour. This can be charged using a square solar panel 25 cm on each side. Assuming an efficiency of 10% and an incident solar power of $1\,\mathrm{kW\,m^{-2}}$ what time is needed to charge the battery?

A 0.10 hours **B** 0.28 hours
C 0.40 hours **D** 1.5 hours [1]

20. A light dependent resistor is connected across an ideal 12 V source and placed in the open in the middle of a desert. When is the power dissipated in the resistor highest?

A dawn **B** mid morning
C noon **D** midnight [1]

21. A bullet with a mass of 10 g is fired at a velocity of $400\,\mathrm{m\,s^{-1}}$ into a cubical tank of water 2 m on each side and is brought to a halt by friction. Given that the heat capacity of water is $4.2\,\mathrm{kJ\,K^{-1}\,kg^{-1}}$, and its density is $1000\,\mathrm{kg\,m^{-3}}$, calculate the temperature rise of the water in the tank

A $2.4 \times 10^{-5}\,\mathrm{K}$ **B** $4.8 \times 10^{-5}\,\mathrm{K}$
C $1.9 \times 10^{-4}\,\mathrm{K}$ **D** $2.4 \times 10^{-2}\,\mathrm{K}$ [1]

22. When using Einstein's formula $E = mc^2$ a student enters the mass m in grams. If he uses a value of $c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$ for the speed of light, what are the units of the energy E?

A mJ **B** J
C kJ **D** MJ [1]

Written answers (20 marks)

23. This problem will consider the possibility of storing electrical energy in capacitors, made up of two parallel metal plates, each of area A, separated by a thickness d of a dielectric with electrical permittivity p. A capacitor will store a charge $q = CV$ where the capacitance is given by $C = pA/d$ and V is the voltage across the capacitor. Given that the work done in charging the capacitor is

$$W = \frac{q^2}{2C}$$

show that the work done to charge a parallel plate capacitor is

$$W = \frac{pAV^2}{2d}.$$

This work done is equal to the energy stored in the capacitor. In practice a capacitor is limited by the *breakdown voltage* of its dielectric which is proportional to the thickness, $V_{\max} = Bd$, where B is a constant that depends on the material. Determine how the maximum energy stored in a capacitor depends on the mass m of the dielectric, its density D and other constants.

For a dielectric with $p = 2 \times 10^{-11}\,\mathrm{F\,m^{-1}}$, $B = 2 \times 10^{7}\,\mathrm{V\,m^{-1}}$, and $D = 1000\,\mathrm{kg\,m^{-3}}$ (which is about right for common plastic dielectrics) calculate the maximum energy can be stored in a capacitor with a mass of 1 kg. Comment on the practicality of using a capacitor of this type to smooth the output from a 1 kW domestic wind turbine in a gusty area. [7]

24. A pilot takes off from an airfield 5 km west of her house and flies in a direction 60° east of north. After 5 minutes she sees that direction to her house is now at angle of 135° to her course. How far away is she from her house? (Use $\sqrt{2} \approx 1.4$) [4]

25. A forest is inhabited by three species of macaw which are all the same shape but are different sizes and colours. The food consumption of each type is proportional to the square of its length. Given that

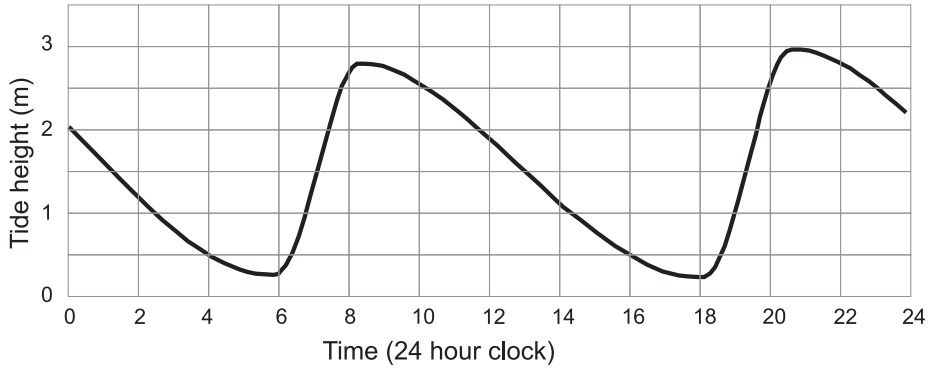
    (a) A crimson macaw and a ruby macaw put together are twice as long as a scarlet macaw

    (b) A crimson macaw and a scarlet macaw put together eat as much as a ruby macaw

    (c) 2 crimson macaws and a scarlet macaw put together are 1 m long

    determine the lengths of the three types of macaw. [6]

26. The graph below shows tide heights at Wadebridge on July 7 2008.



    At what time in the morning (to the nearest hour) is the tide height changing most rapidly? What is the rate of change in cm per minute at this time? [3]

Long question (20 marks)

27. A 2 m tall birdwatcher with a mass of 100 kg sees a nest in a tree. When he stands 18 m away from the tree his line of sight to the nest makes an angle of 45° to a line parallel to the ground.

    (a) The birdwatcher sees an egg fall from the nest. How long does it take to reach the ground? (You may neglect the effects of air resistance.) [3]

    (b) How fast is the egg travelling when it reaches the ground? [2]

    (c) The egg strikes the ground and is brought to a stop in a distance of 1 mm. Assuming a mass of 20 g for the egg calculate the force required. (You may assume a constant braking force.) [3]

    (d) Calculate the work done by this braking force and compare it with the gravitational potential energy of the egg. [3]

    (e) Unsurprisingly the egg is smashed by the impact. To prevent this happening again the birdwatcher places a pad of foam which is 10 cm thick around the tree. This responds to impacts by compressing to half its initial thickness. Find the new braking force when a second egg falls and the time taken to bring the egg to a halt. [3]

    (f) The birdwatcher considers the possibility of returning the egg to the nest by climbing the tree. Calculate the minimum energy he would have to expend to achieve this. [2]

    (g) Instead he decides to use this energy to boil the egg using a small electrical heater powered from a hand generator. Calculate the minimum efficiency required for the system so that it requires no more effort to boil the egg that to return it to the nest. Assume that the egg has a specific heat capacity of $4\,\mathrm{kJ\,kg^{-1}}$, the same boiling point as water, and starts at 20° C. [4]